UNNS and the Dirac Equation: Spinors as Recursive Structures

Abstract

We suggest an interpretation of the Dirac equation through the framework of UNNS (Unbounded Nested Number Sequences). The spinor structure is re-read as recursive doubling, gamma matrices are aligned with UNNS operators, and particle–antiparticle symmetry is understood as recursion echoes. This reframing hints at a deeper substrate unifying discrete recursion and relativistic quantum mechanics.

1 The Dirac Equation

The Dirac equation in relativistic quantum mechanics is

$$(i\gamma^\mu \partial_\mu - m)\psi = 0,$$

where ψ is a four-component spinor, γ^μ are the Dirac gamma matrices, and m is mass.

Its key features include:

- Prediction of antimatter.
- Natural inclusion of spin-$\frac{1}{2}$ degrees of freedom.
- Consistency with special relativity.

2 Spinor Structure as Recursive Doubling

A Dirac spinor has four components, factored as

$$\psi \sim \{\text{spin up}, \text{spin down}\} \times \{\text{particle}, \text{antiparticle}\}.$$

[Recursive Doubling] In the UNNS framework, the Dirac spinor corresponds to a *recursive nest of depth two*, where each level doubles the available states. Thus,

$$\text{Nest}_2 \otimes \text{Nest}_2 \ \cong \ \psi.$$

This captures the essential structure of fermionic degrees of freedom as discrete nested layers.

3 Gamma Matrices as Operators

Gamma matrices connect spinor components. In UNNS, the role of structural operators suggests analogies:

$$\begin{aligned}
\gamma^0 &\leftrightarrow \text{ Collapse operator (time/energy anchor)},\\
\gamma^i &\leftrightarrow \text{ Inlaying operators (spatial lattice embeddings)}.
\end{aligned}$$

Thus, the Dirac operator may be viewed as a *UNNS operator grammar* acting on recursive nests.

4 Antimatter as Echo Symmetry

Dirac's prediction of antiparticles can be reframed as:

- Each recursive layer generates an *echo state*.
- Echoes may invert sign or phase, manifesting as antiparticle symmetry.

This aligns with the UNNS principle that recursion produces both forward and mirrored echoes.

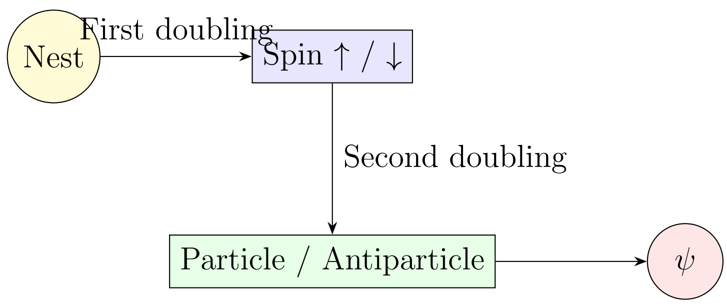
5 Probability Currents and UNNS Flows

The conserved Dirac current,

$$j^\mu = \bar{\psi}\gamma^\mu\psi,$$

represents the flow of information across recursion layers in the UNNS substrate. Conservation expresses stabilization of recursive propagation.

6 Diagram



7 Worked Example: A Toy UNNS–Dirac Spinor Built from Fibonacci

We construct a two-component UNNS spinor whose linear (Dirac-like) update recovers the Fibonacci companion dynamics, and then extend it with UNNS operators (collapse, inlaying) and a discrete gauge coupling (phase inletting).

Spinorization of the Fibonacci Companion

Recall the Fibonacci companion matrix

$$A \;=\; \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \mathrm{spec}(A) = \{\varphi,\, -\varphi^{-1}\}, \quad \varphi = \tfrac{1+\sqrt{5}}{2}.$$

Define the UNNS "spinor" at step n by

$$\Psi_n \;:=\; \begin{bmatrix} a_n \\ a_{n-1} \end{bmatrix} \in \mathbb{R}^2, \qquad \Psi_{n+1} \;=\; A\,\Psi_n.$$

Write the Pauli matrices $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then

$$A \;=\; \tfrac{1}{2}(I + \sigma_z) \;+\; \sigma_x \;=:\; \underbrace{\alpha\,\sigma_x}_{\text{inter-component “hop”}} \;+\; \underbrace{\beta\,\sigma_z}_{\text{imbalance}} \;+\; \underbrace{\gamma\,I}_{\text{drift}},$$

with coefficients $(\alpha, \beta, \gamma) = (1, \frac{1}{2}, \frac{1}{2})$. This yields a *Dirac-like* one-step evolution

$$\Psi_{n+1} \;=\; \big(\alpha\,\sigma_x + \beta\,\sigma_z + \gamma\,I\big)\Psi_n,$$

formally analogous to a discrete-time 1D Dirac update (“hop” σ_x, mass/imbalance σ_z, drift I).

[Dirac-like spectrum equals Fibonacci spectrum] The operator $D := \alpha\,\sigma_x + \beta\,\sigma_z + \gamma\,I$ with $(\alpha, \beta, \gamma) = (1, \frac{1}{2}, \frac{1}{2})$ is similar to A, hence $\operatorname{spec}(D) = \operatorname{spec}(A) = \{\varphi, -\varphi^{-1}\}$. Consequently, for any nonzero seed Ψ_0, the projective direction converges to the φ-eigenline and the effective growth is φ.

[Proof (sketch)] Compute D explicitly to verify $D = A$. Spectral claims follow.

UNNS operators: collapse and inlaying

Introduce UNNS operators acting componentwise:

$$C_\varepsilon(x) = \begin{cases} 0, & |x| < \varepsilon, \\ x, & \text{otherwise}, \end{cases} \qquad G(z) = \operatorname{round}(\Re z) + i\,\operatorname{round}(\Im z).$$

Define the UNNS pipeline

$$\Psi_{n+1} \;=\; \mathcal{T}(\Psi_n) \;:=\; G\big(C_\varepsilon(D\,\Psi_n)\big).$$

Let $\Delta_n := \mathcal{T}(\Psi_n) - D\Psi_n$. Then $\|\Delta_n\|_\infty \le \frac{1}{2}$ (rounding bound), and C_ε acts nontrivially only near the origin.

[Asymptotic spectral stability under UNNS projection] For any fixed $\varepsilon \le \frac{1}{2}$, the *effective growth factor*

$$\lambda_{\text{eff}} \;:=\; \lim_{n\to\infty} \frac{\|\Psi_{n+1}\|}{\|\Psi_n\|}$$

exists and equals φ. Moreover the projective ratio $\Psi_n^{(1)}/\Psi_n^{(2)} \to \varphi$, with error decaying geometrically.

[Proof (outline)] Unroll $\Psi_n = D^n \Psi_0 + \sum_{k=0}^{n-1} D^{n-1-k} \Delta_k$. Since $\|D^m\| \leq C\,\varphi^m$ and $\|\Delta_k\| = O(1)$, the perturbation is $O(\varphi^{n-1})$. Divide by $\|D^n \Psi_0\| \asymp \varphi^n$ to obtain a vanishing relative error. Projective convergence follows from dominance of the φ-eigenline.

Gauge coupling as UNNS inletting (phase connection)

Introduce a discrete $U(1)$ gauge field via a phase inletting on each step:

$$\Psi_{n+1} \;=\; G\big(C_\varepsilon\big(U_{\theta_n}\, D\, \Psi_n\big)\big), \qquad U_{\theta_n} \;=\; \begin{bmatrix} e^{i\theta_n} & 0 \\ 0 & e^{-i\theta_n} \end{bmatrix}.$$

Interpretation:

- U_{θ_n} is a *connection operator* (UNNS inletting) that twists the two components by opposite phases (discrete $U(1)$ charge).
- Constant $\theta_n \equiv \theta$ yields a similarity of D up to a unitary, so the growth spectrum is unchanged: $\lambda_{\text{eff}} = \varphi$.
- Slowly varying θ_n modulates transient interference but leaves the dominant eigenline invariant modulo rounding/repair.

[Spectrum under $U(1)$ inletting] If (θ_n) is bounded and Lipschitz in n (slow twist), then the effective growth remains $\lambda_{\text{eff}} = \varphi$; the phase field affects only finite-time interference and rounding residues.

Physical analogy and UNNS reading

- **Spinor:** two nested degrees ("up/down") realized by the Fibonacci interleaving of (a_n, a_{n-1}).
- **Dirac operator:** $D = \alpha\sigma_x + \beta\sigma_z + \gamma I$, a linear hop+mass drift reproducing Fibonacci spectrum.
- **Measurement/dissipation:** C_ε (collapse) and G (inlaying) act as non-unitary projection/repair while preserving the asymptotic resonance φ.
- **Gauge field:** U_{θ_n} implements a discrete connection (phase inletting); constant fields are spectrally inert, variable fields imprint transient interference patterns.

Numerical signature. Form $T_N := \prod_{k=0}^{N-1} G\, C_\varepsilon\, U_{\theta_k}\, D.$ Then $\sigma_{\max}(T_N)^{1/N} \to \varphi$ and the dominant right singular vector aligns with the φ-eigenline (modulo rounding), confirming the UNNS–Dirac resonance.

Conclusion

The Dirac equation can be reinterpreted in UNNS terms:

- Spinors as recursive nests.
- Gamma matrices as operator grammars.
- Antiparticles as recursion echoes.

This does not replace quantum field theory, but suggests that fermionic structure may ultimately rest upon arithmetic recursion substrates.